Technical Notes

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Optimal Reciprocalization of Measured Displacements—Revisited

Menahem Baruch*
Technion—Israel Institute of Technology, 32000 Haifa, Israel

DOI: 10.2514/1.27877

Introduction

In [1] two methods were proposed for the optimal reciprocalization of measured displacements. In this Note we propose a third method which seems simpler and more elegant.

Procedure

A linear structure must fulfill the Maxwell–Betti reciprocal theorem. This is true for static or dynamic environment ([1–6]). This theorem expressed in matrix form reads

$$F^t Y = Y^t F \tag{1}$$

where $F(n \cdot m)$ is the load matrix and $Y(n \cdot m)$ is the displacement matrix caused by the load matrix applied on the structure. n are the degrees of freedom of the structure and m is the number of loadings. Usually the measured displacements $T(n \cdot m)$, due to different mistakes, do not fulfill the reciprocal theorem. However, for further dynamic or static computations the reciprocal theorem must be fulfilled. The problem is to correct the displacement matrix in an optimal way in which the reciprocal theorem is fulfilled and the corrected displacement matrix $X(n \cdot m)$ is as close as possible to the measured displacement matrix. Here the closeness of two matrices will be ascertained in a Euclidean sense. The only requirement of the load matrix F is to be of rank (m).

Hence,

$$F^t F = f \tag{2}$$

where $f(m \cdot m)$ is a positive definite symmetric matrix. To make the corrected displacements matrix $X(n \cdot m)$ close, in the Euclidean sense, to the measured displacements matrix $T(n \cdot m)$ and to fulfill the Maxwell–Betti reciprocal theorem the following Lagrange function will be used:

$$\begin{aligned}\phi = 1/2\|X - T\| + 1/2\beta|F^t X - X^t F| = 1/2(X_{ij} - T_{ij})^2 \\ + 1/2\beta_{ik}(F_{ji}X_{jk} - X_{ji}F_{jk})\end{aligned} \tag{3}$$

where the Einstein rule for repeated indices was used. Because of the skew-symmetric constrain, the Lagrange multiplier $\beta(m \cdot m)$ is a skew-symmetric matrix,

$$\beta^t = -\beta \tag{4}$$

By equating to zero the first derivative of ϕ with respect to X one obtains

$$X = T - F\beta \tag{5}$$

Now,

$$F^t X = F^t T - f\beta \qquad X^t F = T^t F + \beta f \tag{6}$$

Using the Maxwell–Betti reciprocal theorem, $X^t F = F^t X$, one obtains for β the following matrix equation:

$$f\beta + \beta f = \gamma \tag{7}$$

where γ is a skew-symmetric matrix.

$$\gamma = F^t T - T^t F \tag{8}$$

For more details see [1].

In [1] one can find two different techniques to calculate Eq. (7). We propose here a third technique which seems to be simpler and more elegant.

The matrix f is symmetric and positive definite and hence can be decomposed as follows [7]:

$$A^t A = A A^t = I \qquad A^t f A = \lambda \tag{9}$$

where $A(m \cdot m)$ represents the eigenfunctions and the diagonal matrix $\lambda(m \cdot m)$ represents the positive eigenvalues of f, respectively. Hence, using Eqs. (9) one can rewrite Eq. (7) as follows:

$$\lambda B + B\lambda = G \tag{10}$$

where

$$B = A^t \beta A, \qquad G = A^t \gamma A \tag{11}$$

The solution of Eq. (10) is quite simple:

$$B_{ij} = \frac{G_{ij}}{\lambda_i + \lambda_j} \tag{12}$$

Equation (12) shows that the solution always exists and it is unique.

From Eqs. (11) one obtains finally

$$\beta = A B A^t \tag{13}$$

The corrected displacements can be obtained by using Eq. (5).

In [1] a positive definite symmetric matrix $\psi(5 \cdot 5)$ represents the flexibility matrix of a linear structure. The load matrix $F(5 \cdot 4)$ was chosen quite arbitrarily. The rank of F is, of course, 4 as required. The "measured" displacements $T(5 \cdot 4)$ were obtained by adding to the exact displacements $Y = \psi F$ randomly obtained from ± 1– $\pm$ 5% "errors." Many computations were performed using the numerical example given in [1] and, of course, the results obtained by the three techniques were identical.

Acknowledgment

Thanks to my son Ehud-Moshe Baruch from the Faculty of Mathematics, The Technion, Haifa, for fruitful discussions.

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*Professor Emeritus, Faculty of Aerospace Engineering.

References

[1] Baruch, M., "Optimal Reciprocalization of Measured Displacements," *AIAA Journal*, Vol. 43, No. 10, 2005, pp. 2133–2136.

[2] Sokolnikoff, I. S., *Mathematical Theory of Elasticity*, McGraw–Hill, New York, 1956, p. 391.

[3] Baruch, M., "Methods of Reference Basis for Identification of Linear Dynamic Structures," *Proceedings of the AIAA/ASME/ASCE/AHS, 23rd Structures, Structural Dynamics and Material Conference*, AIAA, New York, 1982, pp. 557–563.

[4] Baruch, M., "Methods of Reference Basis for Identification of Linear Dynamic Structures," *AIAA Journal*, Vol. 22, No. 4, 1984, pp. 561–564.

[5] Baruch, M., Stavrinidis, C., and Fissete, E., "Identification of Fixedly Tested Free Space Structures," ESA, Rept Y/SB/963.89/jb, Noordwijk, The Netherlands, Aug. 1989.

[6] Baruch, M., "Identification of the Damping Matrix," *Contemporary Research in Engineering Mechanics*, edited by G. A. Kardomateas, and V. Birman, American Society of Mechanical Engineers, New York, 2001, pp. 141–160; also American Society of Mechanical Engineers Paper IMECE2001/AD-23763, Nov. 2001.

[7] MATLAB—Mathematical Computing Software, The Math Works, Natick, MA, 2002.

A. Berman
Associate Editor